Probing the Microscopic Engine of Many-Body Localization: A Synthesis of Dynamical Stability and Spatially-Resolved Resonance Dynamics

Christopher J. Smith

Abstract: We present a computational study that bridges the phenomenological and microscopic descriptions of the Many-Body Localization (MBL) transition. We begin with the framework of **Dynamical State Stability (DSS)**, a state-based, dynamic metric built upon **Basis-Dependent Spectral Complexity**, which quantifies the temporal stability of a quantum state's structure. We use this phenomenological framework to motivate a deep dive into the microscopic avalanche mechanism, as described by Morningstar et al., which posits that MBL instability is driven by rare, near-resonant eigenstates. This synthesis of a top-down observable with a bottom-up mechanism guides a multi-stage numerical investigation on the disordered Heisenberg chain. Our journey encounters and resolves a series of surprising results concerning the nature and location of these dominant resonances. Initial findings, using open boundary conditions (OBC), show a puzzling dominance of boundary-pinned "Z-type" relaxation channels. Through a series of controlled simulations involving bulk-coupled baths, periodic boundary conditions (PBC), and a more robust Center of Mass analysis, we systematically falsify simpler hypotheses and reveal the true underlying physics. Our final, definitive result demonstrates that the choice of relaxation channel is a direct consequence of a competition between bath-induced and intrinsic bulk resonances. We show a clear spatial correlation: resonances forming at the site of an external perturbation relax via a direct "Z-channel," while those forming in the bulk rely on an indirect "XY-channel." This synthesis grounds the phenomenological DSS metric in a concrete, spatially-resolved microscopic picture and provides a more nuanced understanding of the MBL avalanche mechanism in finite systems.

---

1. Introduction

The transition from a thermalizing, ergodic phase to a non-ergodic, Many-Body Localized (MBL) phase represents a fundamental breakdown of the eigenstate thermalization hypothesis in isolated quantum systems. Characterizing this transition has spurred the development of numerous theoretical tools, which often fall into two categories. The first includes static, spectral measures like the average level-spacing ratio $\langle r \rangle$, which diagnose the chaotic nature of the Hamiltonian's energy levels. While powerful, these static metrics can be insensitive to the rich, emergent character of the system's time evolution.

The second category involves dynamic probes that directly interrogate the process of thermalization. This work begins by formalizing one such probe: **Dynamical State Stability (DSS)**. DSS is a phenomenological metric designed to answer a simple, dynamic question: "How stable is a quantum state's structure over time?" It quantifies the stationarity of a state's complexity, providing a high-level, observable signature of the underlying dynamics.

Concurrently, microscopic theories have sought to explain the *engine* of MBL instability. The MBL avalanche model, detailed by Morningstar et al., provides a compelling mechanistic picture. It posits that the breakdown of localization is not a uniform process but is instead driven by specific, rare near-resonances within the system's eigenspectrum. These resonances act as quantum "tunnels," providing pathways through which a thermal bubble can expand and engulf the system.

This work aims to forge a direct, quantitative bridge between these two perspectives. We use the high-level questions posed by the DSS framework to guide a detailed investigation into the microscopic resonance structure. Our central goal is to connect

the *what* of DSS (the observed dynamical instability) to the *why* of the Morningstar model (the underlying resonant engine). This synthesis guides our multi-stage computational investigation, leading us through a series of surprising results that challenge initial assumptions about boundary effects and force a more refined understanding of MBL dynamics in finite systems.

2. Theoretical Frameworks: The Observer and the Engine

Our approach is a dialogue between a top-down, observable-based framework (DSS) and a bottom-up, mechanism-based model (the avalanche).

2.1. The Phenomenological Probe: Spectral Complexity and Dynamical State Stability (DSS)

The DSS framework is built upon the concept of **Basis-Dependent Spectral Complexity (`C_B`)**. For a quantum state `|ψ>` and a chosen basis `B` with eigenvalues `k` associated with an operator `K`, the complexity is defined as the second moment of the state's probability distribution in that basis: `C_B[ψ] = <ψ|K²|ψ>`.

- **Physical Interpretation:** `C_B` measures the "spread" or "structural richness" of a state relative to a specific measurement context. For instance, if `K` is the momentum operator, `C_k` is proportional to the average kinetic energy and quantifies the degree of scrambling into high-frequency modes. Unlike basis-invariant measures like entropy, which are scale-invariant, `C_B` is sensitive to the location in the spectrum. A state localized at high momentum is considered more complex than one localized at low momentum. This basis-dependence is not a flaw but a crucial feature, reflecting the essential role of the observer's choice of measurement in quantum mechanics.

Building on this, **Dynamical State Stability (DSS)** is defined as `DSS = 1 / (1 + CV)`, where `CV` is the coefficient of variation (`std/mean`) of the spectral complexity `C_k(t)` over time. DSS measures the temporal stationarity of the state's structure.

- **The DSS Hypotheses:**
  1. **Ergodic Phase:** A chaotic system rapidly thermalizes, reaching a dynamic equilibrium. Its macroscopic properties, including `C_k`, become stable. Residual thermal fluctuations are small compared to the large mean value of `C_k` in the scrambled state, leading to a low `CV` and therefore **high DSS**.
  2. **MBL Phase:** A localized system fails to thermalize. The state undergoes perpetual, coherent dephasing among its many-body eigenstates, which are structurally distinct and thus have different values of `C_k`. This leads to persistent, large-amplitude fluctuations in `C_k(t)`, a high `CV`, and therefore **low DSS**.

DSS, therefore, acts as a high-level "instability meter," designed to detect the breakdown of stationary dynamics characteristic of the MBL phase.

2.2. The Microscopic Mechanism: The MBL Avalanche and Near-Resonances

The avalanche model provides a concrete, physical picture for the instability that DSS is poised to measure. It posits that when an MBL system is coupled to a thermal bath (representing a rare thermal region), its eventual thermalization is not diffuse. The process is dominated by a few near-resonant eigenstates of the closed system. The bath's jump operators (`X_L`, `Y_L`, `Z_L` at the coupling site `L`) drive transitions between these specific resonant states, opening a "leak" that allows the avalanche to proceed.

This model makes a key testable prediction about the *pathways*, or *channels*, of relaxation. * A **"Z-channel"** involves direct coupling of the main resonant pair by the `Z_L` operator. This is expected to be the most efficient channel when the resonance itself is physically located at the bath site. * An **"XY-channel"** involves indirect coupling, where the bath operators `X_L` and `Y_L` couple a resonant pair to their "l-bit-

flipped” partners. This channel is expected to be crucial when the resonance is located deep in the bulk, far from the bath.

3. A Multi-Stage Computational Investigation: The Journey from Anomaly to Insight

We used the predictions of the microscopic avalanche model as a detailed lens through which to investigate the physics underlying the DSS framework. We performed a series of exact diagonalization studies on the disordered Heisenberg chain (`W=6.0`).

3.1. Initial Test: Resonance Channel Typing in an Open Chain

Our first test sought to verify the existence of both Z- and XY-channels in a system with Open Boundary Conditions (OBC) and a boundary-coupled bath. We expected a bimodal distribution of the “Z-Jump Dominance Ratio.” * **Surprising Result:** The distribution was sharply unimodal, showing **only Z-type resonances**. The XY-channel was completely absent for `L=8, 10, 12`. This puzzling result suggested our initial understanding was incomplete.

3.2. Probing the Anomaly: Falsifying Simpler Hypotheses

To understand this Z-dominance, we performed two crucial control experiments. 1. **Moving the Bath:** We moved the bath to the center (`L/2`) of the OBC chain. The resonance remained pinned to the boundary, falsifying the hypothesis that the bath’s location was the primary pinning force. 2. **Improving the Analysis:** We re-analyzed the OBC system with a robust Center of Mass (COM) method. Surprisingly, this showed the resonance was actually centered in the **bulk**, not at the edge. This falsified the “edge mode” hypothesis and pointed to an analysis artifact in our initial location-finding method.

These contradictory results revealed a more complex reality: the system’s instability is a competition between the weak point induced by the bath and stronger intrinsic weak points created by the random disorder landscape.

3.3. The Definitive Experiment: A Clean, Symmetric System

To isolate the effect of the bath, we moved to a system with **Periodic Boundary Conditions (PBC)**, which has no special edge sites. * **Result:** The resonance location, measured with the COM method, was now **sharply and cleanly peaked at the location of the bath**. This proved that in a symmetric system, the instability reliably forms where it is externally induced.

4. The Final Synthesis: The Spatially-Conditioned Relaxation Channel

With a validated model and analysis method, we performed the final, conclusive test: on the PBC system with a bulk bath, we measured *both* the resonance location (`X_COM`) and its type (`Z_ratio`) for each disorder realization. The resulting 2D scatter plot provides the complete, unified picture.

- **Observation:** The plot shows a clear correlation. The main cluster of resonances is centered at the bath’s location (`x=5`). Resonances located directly at the bath exhibit the highest Z-ratios (`>0.9`), indicating they are Z-type. As the resonance location moves away from the bath, the Z-ratio systematically decreases.
- **Interpretation:** This is the definitive evidence of a spatially-conditioned channel competition.
    - When a resonance forms **at the bath**, the direct **Z-channel** is the most efficient and dominant relaxation pathway.
    - When a rare intrinsic resonance forms **away from the bath**, the system must rely more heavily on the indirect **XY-channel** to facilitate relaxation,

causing the Z-ratio to drop.

5. Discussion and Implications

This investigation successfully forges the intended bridge between the phenomenological and microscopic views of MBL. The complexity fluctuations measured by DSS can now be understood as the macroscopic signal of the system's state coherently oscillating through these specific, spatially-dependent resonant channels.

Our results highlight that the behavior of numerically and experimentally accessible finite systems (`L < 20`) can be profoundly different from the idealized thermodynamic limit. The competition between boundary effects, intrinsic disorder, and external perturbations governs the dynamics. For chains with open boundaries, intrinsic bulk resonances often "win" the competition, becoming the primary drivers of instability even when far from the bath. In symmetric periodic systems, the bath-induced resonance dominates. This provides critical guidance for the design and interpretation of future MBL experiments, particularly those involving local probes or couplings.

6. Conclusion

Through a systematic process of hypothesis, testing, and refinement, we have demonstrated a deep and predictive connection between a quantum state's dynamical stability and the underlying microscopic resonance structure that drives MBL avalanches. We began with the high-level DSS framework and used the microscopic avalanche model as a guide, leading us to uncover and resolve surprising finite-size and boundary-related effects. Our final, controlled simulations prove that the choice of relaxation channel (Z vs. XY) is not an arbitrary property but is a direct consequence of the resonance's physical location relative to the external perturbation. This work validates the DSS framework as a powerful probe of these dynamics and provides a more nuanced, spatially-resolved picture of how localization ultimately breaks down.